A Kinetic Theory Approach to Quantum Gravity[1]

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We describe a kinetic theory approach to quantum gravity by which we mean a theory of the microscopic structure of space-time, not a theory obtained by quantizing general relativity. A figurative conception of this program is like building a ladder with two knotty poles: quantum matter field on the right and space-time on the left. Each rung connecting the corresponding knots represents a distinct level of structure. The lowest rung is hydrodynamics and general relativity; the next rung is semiclassical gravity, with the expectation value of quantum fields acting as source in the semiclassical Einstein equation. We recall how ideas from the statistical mechanics of interacting quantum fields helped us identify the existence of noise in the matter field and its effect on metric fluctuations, leading to the establishment of the third rung: stochastic gravity, described by the Einstein–Langevin equation. Our pathway from stochastic to quantum gravity is via the correlation hierarchy of noise and induced metric fluctuations. Three essential tasks beckon: (1) deduce the correlations of metric fluctuations from correlation noise in the matter field; (2) reconstituting quantum coherence—this is the reverse of decoherence—from these correlation functions; and (3) use the Boltzmann–Langevin equations to identify distinct collective variables depicting recognizable metastable structures in the kinetic and hydrodynamic regimes of quantum matter fields and how they demand of their corresponding space-time counterparts. This will give us a hierarchy of generalized stochastic equations—call them the Boltzmann–Einstein hierarchy of quantum gravity—for each level of space-time structure, from the the macroscopic (general relativity) through the mesoscopic (stochastic gravity) to the microscopic (quantum gravity).

KEY WORDS: quantum gravity; stochastic fields; kinetic theory.

1. INTRODUCTION

In the last decade a statistical mechanics description of particles, fields and space-time based on the concept of quantum open systems and the influence functional formalism has been introduced. It reproduces in full the established theory of quantum fields in curved space-time (Birrell and Davies, 1982; Dewitt, 1975; Fulling, 1989; Grib *et al.*, 1994; Mirzabekian and Vilkovisky, 1998; Wald, 1994)

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and contains also a microscopic description of their stochastic properties, such as noise, fluctuations, decoherence, and dissipation. This new framework allows one to explore the quantum statistical properties of space-time beyond the semiclassical regime, as well as important nonequilibrium processes in the early universe and black holes, such as particle creation, entropy generation, structure formation, Hawking radiation, horizon fluctuations, backreaction and the black hole information "loss" issues. This theory describing particles and fields is known as statistical, stochastic, or kinetic field theory, while that applied to space-time dynamics defines the stochastic gravity program.[3]

The search on the gravity side proceeded from the classical level described by Einstein's general relativity theory to the semiclassical level described by the semiclassical Einstein equation. (This is semiclassical gravity, or curved space-time quantum field theory with backreaction (Calzetta and Hu, 1987; Campos and Verdaguer, 1994).) Stochastic gravity is at the next higher level. The progression on the matter side is better known. The classical matter is usually described by a hydrodynamic equation of state. At the semiclassical level it is given by the expectation value of the energy momentum tensor operator of matter fields with respect to some quantum state. We have the microscopic theory of ordinary matter—QED and QCD—so it is easy to deduce its meso and macro forms. On the gravity side it goes the other way. We have the macrotheory, Einstein's general relativity. In our view general relativity is the hydrodynamic regime of the fundamental theory, with the metric and the connection forms as the collective variables in this long wavelength regime, which are likely to lose their meaning and usefulness as we probe into much shorter scales. We want the microscopic theory of space-time structure and dynamics.[4]

Our strategy is to look closely into the quantum and statistical mechanical features of the matter field in deepening levels (This is different from the induced gravity program (Sakharov, 1982) although the spirit is similar.)[5] and see

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[3] One can find a sample of original papers leading to the establishment of this field in (Calzetta and Hu, 1994; Hu and Matacz, 1995; Hu and Sinha, 1995; Campos and Verdaguer, 1996; Martin and Verdaguer, 1999a; Hu and Phillips, 2000) and recent reviews in (Hu, 1999; Hu and Verdaguer, 2001) (the first is mainly on ideas, the second is technical). On statistical field theory, one can find the material most relevant to our discussions here, i.e., the correlation hierarchy, master effective action, decoherence of correlation history, and stochastic Boltzmann equation in (Calzetta 1988; Calzetta and Hu, 1993, 1995, 2000).

[4] This theory is usually referred to as quantum gravity, but it does not mean that quantizing the metric or the connection functions will lead to this microtheory. In fact if these were the hydrodynamic collective variables, doing so will only give us a theory describing the quantized modes of hydrodynamic excitations, not about the microscopic structure of space-times (Hu, 1996a). (A similar viewpoint is expressed by Jacobson (1995) from a different angle.)

[5] I made this choice two decades ago because the geometric structure (left hand side) being made of marble, according to Einstein, is beautiful to look at but difficult to chisel into, whereas the sandstone structure (right hand side) of matter, though not as elegant, is more malleable and easier to work with.

what this implies on the space-time structure at the corresponding levels. Thus we work on the quantum matter field from both ends of its structure, the microstructure described by quantum field theory and the macrostructure described by hydrodynamics. In statistical physics it is well known that intermediate regimes exist between the long-wavelength hydrodynamics limit and the microdynamics[6] Following this pathway we need to perform *three tasks*: First, understand the statistical mechanics of interacting quantum fields in the matter sector. Second, find out from statistical mechanical considerations if there is a corresponding ordering of structure in the gravity sector, starting with Einstein's general relativity not just as geometro-dynamics (Wheeler, 1962) but as geometro-hydrodynamics (Hu, 1996a). Third, construct such a structure for gravity leading to the microtheory of space-time. A figurative conception is that we are given two knotted vertical poles representing space-time and matter to build a ladder, with each rung connecting the corresponding knots on two sides representing a distinct level of structure. The lowest rung appearing to us, creatures living at low energy, is hydrodynamics; the next rung is semiclassical gravity, the third, stochastic gravity. Stochastic gravity actually entails all the higher rungs between semiclassical and quantum gravity, much like the BBGKY or the Dyson–Schwinger hierarchy representing kinetic theory of matter fields. One should then ask: What are the salient features of this kinetic theory regime of space-time structure? What can it reveal about the microscopic theory of space-time structure? The threads sustaining both the vertical and horizontal structures in this conceptual ladder are *two basic issues*: micro/macro interface and quantum/classical correspondence. That is why mesoscopic physics also enters into our consideration (Hu, 1995) in a central way.

Let me indicate three signals flying across these two vertical poles of the ladder which inspired us to the construction of the stochastic gravity theory. (1) *Dissipation*: Our work in the 1980s on backreaction of particle creation from quantum matter fields in cosmological space-times or background fields showed the appearance of dissipation in the background space-time or field dynamics (Calzetta and Hu, 1987, 1989; Campos and Verdaguer, 1994). (2) *Noise and fluctuation*: In trying to understand the statistical mechanical meaning of this dissipation we were led to the understanding that there should be a noise term arising from the fluctuations of the quantum matter field (Hu, 1989) and correspondingly there should be an equation capturing the effect of noise on metric fluctuations: the Einstein–Langevin equation (ELE) (Calzetta and Hu, 1994; Campos and Verdaguer, 1996; Hu and Matacz, 1995; Hu and Sinha, 1995; Lombardo and Mazzitelli, 1997), which forms the centerpiece of stochastic gravity (Hu, 1999; Hu and Verdaguer, 2001; Martin and Verdaguer, 1999). (3) *Decoherence and stochasticity*: One important

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[6] They are usually lumped together and called the kinetic regime, but I think there must be distinct kinetic collective variables depicting recognizable metastable intermediate structures in this vast interim regime (see Spohn (1991) for a more fine-grained description).

understanding which came out of research on quantum to classical transition (which began in the late 1980s for us working on quantum cosmology) was to recognize that a new stochastic regime necessarily lie between the semiclassical and the quantum in almost all physical processes. Noise is instrumental to decoherence in the transition from quantum to classical, resulting in a classical stochastic dynamics (Gell-Mann and Hartle, 1990, 1993; Hu, 1991; Hu *et al.*, 1992).

The question we wish to focus on here is how to relate the stochastic regime to the quantum regime, first for matter field and then for space-time geometry. We first give a sketch of the two poles here: stochastic gravity and kinetic field theory.

1.1. From Semiclassical to Stochastic Gravity

In semiclassical gravity the classical space-time (with metric g_{ab}) is driven by the expectation value $\langle \quad \rangle$ of the stress energy tensor T_{ab} of a quantum field with respect to some quantum state. One main task in the 1970s was to obtain a regularized expression for this quantum source of the semiclassical Einstein equation (SCE). In stochastic gravity of the 1990s the additional effect of fluctuations of the stress energy tensor (Ford, 1982; Hu and Philips, 2000; Philips and Hu, 1997) enters which induces metric fluctuations in the classical space-time described by the ELE (Calzetta and Hu, 1994; Campos and Verdaguer, 1996; Hu and Matacz, 1995; Hu and Sinha, 1995). This stochastic term measures the fluctuations of quantum sources (e.g., arising from the difference of particles created in neighboring histories) and is intrinsically linked to the dissipation in the dynamics of space-time by a fluctuation–dissipation relation which embodies the full backreaction effects of quantum fields on classical space-times.

The stochastic semiclassical Einstein equation, or ELE, takes on the form

$$G_{ab}[g] + \Lambda g_{ab} = 8\pi G(T_{ab}^c + T_{ab}^{qs})$$
$$T_{ab}^{qs} \equiv \langle T_{ab} \rangle_q + T_{ab}^s \tag{1.1}$$

where G_{ab} is the Einstein tensor associated with g_{ab} and Λ, G are the cosmological and Newton constants respectively. Here we use the superscripts c, s, q to denote classical, stochastic, and quantum, respectively. The new term $T_{ab}^s = 2\tau_{ab}$, which is of classical stochastic nature, measures the fluctuations of the energy momentum tensor of the quantum field. To see what τ_{ab} is, first define

$$\hat{t}_{ab}(x) \equiv \hat{T}_{ab}(x) - \langle \hat{T}_{ab}(x) \rangle \hat{I}, \tag{1.2}$$

which is a tensor operator measuring the deviations from the mean of the stress energy tensor in a particular state. We are interested in the correlation of these operators at different space-time points. Here we focus on the stress energy (operator-valued) bitensor $\hat{t}_{ab}(x)\hat{t}_{c'd'}(y)$ defined at nearby points (x, y). A bitensor is a geometric object that has support at two separate space-time points. In particular, it is

a rank 2 tensor in the tangent space at x (with unprimed indices) and in the tangent space at y (with primed indicies).

The noise kernel $N_{abc'd'}$ bitensor is defined as

$$4N_{abc'd'}(x, y) \equiv \frac{1}{2}\langle\{\hat{t}_{ab}(x), \hat{t}_{c'd'}(y)\}\rangle, \tag{1.3}$$

where $\{\ \}$ means taking the symmetric product. In the influence functional (IF) (Feynman and Vernon, 1963) or closed-time-path (CTP) effective action approach (Bakshi and Mahanthappa, 1963; Calzetta and Hu, 1987; Campos and Verdaguer, 1994; DeWitt, 1986; Jordan, 1986; Keldysh, 1964; Schwinger, 1963; Su *et al.*, 1988; Zhou *et al.*, 1985) the noise kernel appears in the real part of the influence action. The noise kernel defines a real classical Gaussian stochastic symmetric tensor field τ_{ab}, which is characterized to lowest order by the following relations,

$$\langle\tau_{ab}(x)\rangle_s = 0, \quad \langle\tau_{ab}(x)\tau_{c'd'}(y)\rangle_s = N_{abc'd'}(x, y), \tag{1.4}$$

where $\langle\ \rangle_s$ means taking a statistical average with respect to the noise distribution τ (for simplicity we do not consider higher order correlations). Since $\hat{T}_{ab}$ is self-adjoint, one can see that $N_{abc'd'}$ is symmetric, real, positive and semidefinite. Furthermore, as a consequence of (1.3) and the conservation law $\nabla^a\hat{T}_{ab} = 0$, this stochastic tensor τ_{ab} is divergenceless in the sense that $\nabla^a\tau_{ab} = 0$ is a deterministic zero field. Also $g^{ab}\tau_{ab}(x) = 0$, signifying that there is no stochastic correction to the trace anomaly for massless conformal fields where T_{ab} is traceless (see Martin and Verdaguer (1999)). Here all covariant derivatives are taken with respect to the background metric g_{ab}, which is a solution of the semiclassical equations. Taking the statistical average of (1.1) with respect to the noise distribution τ, as a consequence of the noise correlation relation (1.4),

$$\langle T_{ab}^{qs}\rangle_s = \langle T_{ab}\rangle_q \tag{1.5}$$

we recover the semiclassical Einstein equation, which is (1.1) without the T_{ab}^s term. It is in this sense that we view semiclassical gravity as a mean field theory.

1.2. Kinetic Theory of Interacting Quantum Fields

Our viewpoint here is motivated by classical kinetic theory. In the dynamics of a dilute gas (Akhierzer and Peletminsky, 1981; Balescu, 1975; Spohn, 1991) the exact Newton's or Hamilton's equations for the evolution of a many body system may be represented by a Liouville equation for the total distribution function or the BBGKY hierarchy for the partial (n particle) distributions. This reformulation is only formal, which involves no loss of information or predictability. Physical description of the dynamics comes from truncating the BBGKY hierarchy and introducing a causal factorization condition ("slaving") whereby the higher order correlations are substituted by functionals of the lower order ones with some

causal boundary conditions (like the molecular chaos assumption) ingrained, which accounts for the appearance of dissipation and an arrow of time in the Boltzmann equation.

For illustrative purpose here we can represent quantum gravity as an interacting quantum field (of fermions?) and we shall traverse this passage using the *correlation dynamics* from the *master effective action*. The master effective action is a functional of the whole string of Green functions of a field theory whose variation generates the Schwinger–Dyson hierarchy. There are *two aspects* in this problem: coherence of a field as measured by its correlation (for quantum as well as classical) and quantum to classical transition. We wish to treat both aspects with a quantum version of the correlation (BBGKY) hierarchy, the Schwinger–Dyson equations. There are *three steps* involved: First, show how to derive the kinetic equations from quantum field theory—or to go from Dyson to Boltzmann. Second, show how to introduce the open system concept to the hierarchy. For this we need to introduce the notion of "slaving" in the hierarchy, which renders a subset made up of a finite number of lower order correlation functions as an effectively open system, where it interacts with the environment made up of the higher order correlation functions. Third, show why there should be a stochastic term in the Boltzmann equation when influence of the higher correlation functions are included.

The first step was taken in the mid-1980s, when, amongst many authors (see Bezzerides and Bois (1972); Calzetta and Hu (1988); Danielewicz (1984); DuBois (1972); Kadanoff and Baym (1962); Li and McLerran (1983); van Leeuwen *et al.* (1980) for earlier work and Abe *et al.* (1996); Boyanovsky *et al.* (1996). Blaizot and Iancu (2001); Calzetta and Hu (2000); Calzetta, Hu, and Ramsey (2000); Henning (1995); Mrowczynski (1997); Mrowczynski and Danielewicz (1990); Mrowczynski and Heinz (1994); Witte *et al.* (1996); Zhuang and Heinz (1998) for recent developments), Calzetta and Hu (1988), showed how the quantum Boltzmann equation arises as a description of the dynamics of quasiparticles in the kinetic limit of quantum field theory. The main element in the description of a nonequilibrium quantum field is its Green functions, whose dynamics is given by the Dyson equations. For the second step, we showed (Calzetta and Hu, 1995) how the coarse-grained (truncation with slaving) n-point correlation functions behave like an effectively open system. It is easy to illustrate this idea with the lowest order elements in the Schwinger–Dyson hierarchy of correlation functions, consisting of the mean field and the two-point function. They are deducible from the CTP 2PI (two-particle-irreducible) effective action which has been used to derive the Boltzmann equation in kinetic field theory, for problems in critical dynamics and many nonequilibrium quantum field processes. Generalizing from $n = 2$ to ∞ yields the master effective action (MEA). In general the full MEA is required to recover full (including phase) information in a quantum field. If we now view the problem in this framework, we can see how dissipation and fluctuations arise

when the hierarchy is truncated and the higher correlations are slaved (we refer to these two procedures combined as coarse-graining), in the same way how Boltzmann equation is derived from the BBGKY hierarchy. In (Calzetta and Hu, 1995) we (1) gave a formal construction of the master effective action, (2) showed how truncation in nPI is related to loop expansion, and (3) how "slaving" leads to dissipation.

Now for the third step: our assertion is that there should also be a noise term present as source in addition to the collision term in the Boltzmann equation, making it a stochastic Boltzmann, or Boltzmann–Langevin (BL) equation. Can we find the noise in the correlation hierarchy? Note that we are now following the Boltzmann paradigm of effectively open systems, not the Langevin paradigm of stipulated open systems. It is more difficult to find the noise from the BBGKY hierarchy (an example for classical gas is that of Kac and Logan (1976, 1979) or the Schwinger-Dyson series of correlation functions of interacting quantum fields than finding noise in a well-defined environment, (e.g., *a la* Feynman–Vernon (1963)). We were partially successful in identifying such a correlation noise (Calzetta and Hu, 2000) arising from the slaving of the higher correlation functions and proving a fluctuation–dissipation relation for these correlation noises. With the BL equation one can use the correlation hierarchy to infer the quantum microdynamics. From the self-consistency between the matter (quantum field) and the space-time as suggested by Einstein's equation, one can construct a parallel hierarchy of metric correlation functions induced by the matter field. From this hierarchy, beginning with stochastic gravity, one could begin to unravel the microstructure of space-time.

This summarizes the two theories which make up the poles of our ladder. We will now try to apply the statistical mechanical ideas to find a route from stochastic to quantum gravity. In Section 2 we give a sketch of the stochastic regime in relation to the semiclassical and quantum regimes, with the specific aim of seeking a pathway to quantum gravity. In Section 3 we highlight the issues in the statistical mechanics of interacting quantum fields which we need to address. In Section 4 we give an example of how the correlation hierarchy of quantum fields can be applied to a possible resolution of the black hole information loss paradox, on the basis of ideas proposed before. Finally in Section 5 we develop further some issues anticipated en route, i.e., the reconstruction of coherence via the correlation hierarchy of space-time fluctuations. This involves ideas from kinetic field theory and mesoscopic transport theory. We end with a task summary.

2. STOCHASTIC IN RELATION TO SEMICLASSICAL AND QUANTUM GRAVITY

We see that stochastic semiclassical gravity provides a relation between noise in quantum fields and metric fluctuations. While the semiclassical regime describes

the effect of a quantum matter field only through its mean value (e.g., vacuum expectation value), the stochastic regime includes the effect of fluctuations and correlations. We believe that precious new information resides in the two-point functions of the stress energy tensor which may reflect the finer structure of space-time at a scale when information provided by its mean value as source (semiclassical gravity) is no longer adequate. To appreciate this, it is perhaps instructive to examine the distinction among these three theories: stochastic gravity in relation to semiclassical and quantum gravity (Hu, 1999; Martin and Verdaguer, 1999). The following observation (adopted from Hu (1999) will also bring out two other related concepts of correlation (in the quantum field) and coherence (in quantum gravity).

2.1. Classical, Stochastic, and Quantum

For concreteness we consider the example of gravitational perturbations h_{ab} in a background space-time with metric g_{ab} driven by the expectation value of the energy momentum tensor of a scalar field Φ, as well as its fluctuations $\hat{t}_{ab}(x)$. Let us compare the stochastic with the semiclassical and quantum equations of motion for the metric perturbation (weak but deterministic) field h. (This schematic representation was made by E. Verdaguer in Peyresq 3.) The semiclassical equation is given by

$$\Box h = 16\pi G \langle \hat{T} \rangle, \tag{2.1}$$

where $\langle \rangle$ denotes taking the quantum average (e.g., the vacuum expectation value) of the operator enclosed. Its solution can be written in the form

$$h = \int C \langle \hat{T} \rangle, \qquad h_1 h_2 = \int \int C_1 C_2 \langle \hat{T} \rangle \langle \hat{T} \rangle. \tag{2.2}$$

The quantum (Heisenberg) equation

$$\Box \hat{h} = 16\pi G \hat{T} \tag{2.3}$$

has solutions

$$\hat{h} = \int C \hat{T}, \qquad \langle \hat{h}_1 \hat{h}_2 \rangle = \int \int C_1 C_2 \langle \hat{T} \hat{T} \rangle_{\hat{h}, \hat{\phi}}, \tag{2.4}$$

where the average is taken with respect to the quantum fluctuations of both the gravitational ($\hat{g}$) and the matter ($\hat{\phi}$) fields. Now for the stochastic equation, we have

$$\Box h = 16\pi G(\langle \hat{T} \rangle + \tau), \tag{2.5}$$

with solutions[7]

$$h = \int C \langle \hat{T} \rangle + \int C \tau, \qquad h_1 h_2 = \int \int C_1 C_2 [\langle \hat{T} \rangle \langle \hat{T} \rangle + (\langle \hat{T} \rangle \tau + \tau \langle \hat{T} \rangle) + \tau \tau].$$

(2.6)

Now take the noise average $\langle \rangle_\tau$. Recall that the noise is defined in terms of the stochastic sources τ as

$$\langle \tau \rangle_\tau = 0, \qquad \langle \tau_1 \tau_2 \rangle_\tau \equiv \langle \hat{T}_1 \hat{T}_2 \rangle - \langle \hat{T}_1 \rangle \langle \hat{T}_2 \rangle,$$

(2.7)

we get

$$\langle h_1 h_2 \rangle_\tau = \int \int C_1 C_2 \langle \hat{T} \hat{T} \rangle_{\hat{\phi}}.$$

(2.8)

Note that the correlation of the energy momentum tensor appears just like in the quantum case, but the average here is over noise from quantum fluctuations of the matter field alone.

2.2. Noise and Fluctuations, Correlations and Coherence

Comparing the equations above depicting the semiclassical, stochastic, and quantum regimes, we see first that in the semiclassical case, the classical metric correlation is given by the product of the vacuum expectation value of the energy momentum tensor whereas in the quantum case it is given by the quantum average of the correlation of metric (operators) with respect to the fluctuations in both the matter and the gravitational fields. In the stochastic case the form is closer to the quantum case except that the quantum average is replaced by the noise average, and the average of the energy momentum tensor is taken with respect only to the matter field. The important improvement over semiclassical gravity is that it now carries information on the correlation of the energy momentum tensor of the fields and its induced metric fluctuations. Thus stochastic gravity contains information about the correlation of fields (and the related phase information) which is absent in semiclassical gravity. Here we have invoked the relation between *fluctuations* and *correlations*, a variant form of the fluctuation-dissipation relation. This feature moves stochastic gravity closer than semiclassical gravity to quantum gravity in that the correlation in quantum field and geometry fully present in quantum gravity

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[7] In this schematic form we have not displayed the homogeneous solution carrying the information of the (maybe random) initial condition. This solution will exist in general, and may even be dominant if dissipation is weak. When both the uncertainty in initial conditions and the stochastic noise are taken into account, the Einstein–Langevin formalism reproduces the exact graviton two-point function, in the linearized approximation. Of course, it fails to reproduce the expectation value of observables which could not be written in terms of graviton occupation numbers, and it is in these aspects which one needs to reconstruct to get to full quantum gravity. This comment was made by E. Calzetta in a correspondence to the author.

is partially retained in stochastic gravity, and the background geometry has a way to sense the correlation of the quantum fields through the noise term in the ELE, which shows up as metric fluctuations.

By now we can see that "noise" as used in this more precise language and broader context is not something one can arbitrarily assign or relegate, as is often done in ordinary discussions, but it has taken on a deeper meaning in that it embodies the contributions of the higher correlation functions in the quantum field. It holds the key to probing the quantum nature of space-time in this vein. We begin our studies here with the lowest order term, i.e., the two-point function of the energy momentum tensor which contains the fourth-order correlation of the quantum field (or gravitons when they are considered as matter source).[8] Progress is made now on how to characterize the higher order correlation functions of an interacting field systematically from the Schwinger-Dyson equations in terms of "correlation noise" (Calzetta and Hu, 1995, 2000), after the BBGKY hierarchy. This will prove to be useful for a correlation dynamics/stochastic semiclassical approach to quantum gravity (Hu, 1999).

Thus noise carries information about the correlations of the quantum field. One can further link *correlation* in quantum fields to *coherence* in quantum gravity. This stems from the self-consistency required in the backreaction equations for the matter and space-time sectors. The ELE is only a partial (low energy) representation of the complete theory of quantum gravity and fields. There, the coherence in the geometry is related to the coherence in the matter field, as the complete quantum description should be given by a coherent wave function of the combined matter and gravity sectors. Semiclassical gravity forsakes all the coherence in the quantum gravity sector. Stochastic gravity captures only partial coherence in the quantum gravity sector via the correlations in the quantum fields. Since the degree of coherence can be measured in terms of correlations, our strategy for the semiclassical stochastic gravity program is to unravel the higher correlations of the matter field, going up the hierarchy starting with the variance of the stress energy tensor, and through its linkage with gravity (the lowest rung provided by the Einstein equation), retrieve whatever quantum attributes (partial coherence) of gravity left over from the high energy behavior above the Planck scale. Thus in this approach, focusing on the noise kernel and the stress energy tensor two-point function is our first step beyond mean field (semiclassical gravity) theory towards probing the full theory of quantum gravity.

In recent years there has been increasing attention paid to the effects of a fluctuating space-time such as related to structure formation (Calzetta and Gonorazky,

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[8] Although the Feynman–Vernon way can only accomodate Gaussian noise of the matter fields and takes a simple form for linear coupling to the background space-time, the notion of noise can be made more general and precise. For an example of more complex noise associated with more involved backreactions arising from strong or nonlocal couplings, see Johnson and Hu (2002).

1997; Calzetta and Hu, 1995; Matacz, 1997; Roura and Verdaguer, 1999, 2000), black hole event horizons and Hawking radiation (Barrabès and Parentani, 2000; Campos and Hu, 1998; Candelas and Sciama, 1977; Ford and Svaiter, 1977; Hu *et al.*, 1998; Hu and Shiokawa, 1998; Massar and Parentani, 2000; Mottola, 1986; Sorkin, 1996; Niemeyer and Parentani, 2001; Parentani, 2001; Wu and Ford, 1999), and space-time foam (Carlip, 1997, 1998; Garay, 1998, 1999). Only a small subset of work has included backreaction considerations, such as that of induced metric fluctuations from a quantum scalar in Minkowski space-time treated by Martin and Verdaguer (2000), which is closest to the spirit of our description above. Better understanding in the relation of quantum correlation functions with its classical (Anastopoulos, 2001) and stochastic (Calzetta *et al.*, 2001) counterparts is also conducive to the pursuit of our program.

To continue this line of thought to seek a pathway to the microstructure of space-time (our definition of quantum gravity) via the correlation hierarchy, we need some more discussions on the statistical mechanics of quantum fields.

3. STATISTICAL MECHANICS OF AN INTERACTING QUANTUM FIELD

Although interacting quantum fields is a familiar subject which one learns in the first lessons of quantum field theory, not much has been explored in its statistical mechanical content. It is surprisingly rich, much like the role an ordinary box of gas molecules plays in Boltzmann's sophisticated theory of kinetics, dissipation, and arrow of time. In Section 1.2 we mentioned the three steps taken to obtain a stochastic Boltzmann equation and the two aspects: The micro–macro relation and the quantum to classical transition. Here we expand on those two aspects (details can be found in Calzetta and Hu (1988, 1993, 1995, 2000)).

The statistical mechanical properties of interacting quantum fields can be studied in terms of the *dynamics of the correlation functions*. The full dynamics of an interacting quantum field may be described by means of the Dyson-Schwinger equations governing the infinite hierarchy of Wightman functions which measure the correlations of the field. This hierarchy of equations can be obtained from the variation of the infinite particle irreducible, or *"master" effective action* (MEA). Truncation of this hierarchy gives rise to a quantum subdynamics governing a finite number of correlation functions (which constitute the "system"). "Slaving" refers to expressing the higher order correlation functions (which constitute the "environment") in terms of the lower order ones by functional relations such as the "factorization" and the imposition of causal conditions ("molecular chaos" assumption). The latter condition induces *dissipation* in the dynamics of the sub-system. In addition to the collision integrals there should also be a stochastic source representing the fluctuations of the environment, which we call the *"correlation*

noise." We posit that at each level of the hierarchy these two aspects should be related by a fluctuation–dissipation relation.

This is the quantum field equivalent of the BBGKY hierarchy in Boltzmann's theory. Any subsystem involving a finite number of correlation functions defines an effective theory. The relation of loop expansion and correlation order was expounded (Calzetta and Hu, 1995). We see that ordinary quantum field theory which involves only the mean field and a two-point function, or any finite-loop effective action in a perturbative theory are, by nature, also effective theories. Histories defined by lower order correlation functions can be decohered by the noises from the higher order functions and acquire quasi-classical stochastic attributes. We think this scheme invoking the correlation order is a natural way to describe the quantum to classical transition for a closed system as it avoids ad hoc stipulation of the system–environment split. It is in this spirit that our kinetic theory approach to quantum gravity works.

3.1. Statistical Mechanics Aspect: Correlation Dynamics in the BBGKY or Dyson–Schwinger Hierarchy

So we want to describe an interacting quantum field in terms of the (infinite number of) correlation functions. In the consistent history approach (Griffiths, 1984; Omnés, 1988, 1990, 1992) to quantum mechanics, we view the "mean" field not as the actual expectation value of the field, but rather as representing the local value of the field within one particular history. Quantum evolution encompasses the coherent superposition of all possible histories and these quantities are subject to fluctuations. This naturally introduces into the theory stochastic elements, which has hitherto been largely ignored in the usual description of quantum field theory. The usual loop expansion in quantum field theory is replaced in our scheme by the correlation functions up to a certain order acting as independent variables, along with the "mean" field, which themselves are subject to fluctuations.

Our starting point is the well-known fact that the set of all Wightman functions (time ordered products of field operators) determines completely the quantum state of a field (Haag, 1992). Instead of following the evolution of the field in any of the conventional representations (Schrödinger, Heisenberg, or Dirac's), we focus on the dynamics of the full hierarchy of Wightman functions. To this end it is convenient to adopt Schwinger's CTP techniques (Bakshi and Mahanthappa, 1963; Keldysh, 1964; Schwinger, 1963; Su *et al.*, 1988; Zhou *et al.* 1985), and consider time ordered Green functions as a subset of all Green functions path—ordered along a closed time loop. The dynamics of this larger set is described by the Dyson-Schwinger equations.

We first showed that the Dyson-Schwinger hierarchy may be obtained via the variational principle from a functional which we call the MEA. This is a formal action functional where each Wightman function enters as an independent variable.

We then showed that any field theory based on a finite number of (mean field plus) correlation functions can be viewed as a subdynamics of the Dyson-Schwinger hierarchy. The specification of a subdynamics involves two steps: First, the hierarchy is *truncated* at a certain order. A finite set of variables, say, the lowest nth order correlation functions, is identified to be the "relevant" (Mori, 1965; Nakajima, 1958; Willis and Picard, 1974; Zwanzig, 1960) variables, which constitute the subsystem. Second, the remaining "irrelevant" or "environment" variables, say, the $n + 1$ to ∞ order correlation functions, are *slaved* to the former. Slaving (imposition of the factorization and causal conditions) means that irrelevant variables are substituted by set functionals of the relevant variables. The process of extraction of a subdynamics from the Dyson-Schwinger hierarchy shows up at the level of the effective action, where the MEA is truncated to a functional of a finite number of variables. The finite effective actions so obtained (the influence action (Caldeira and Leggett, 1983; Feynman and Hibbs, 1965; Feynman and Vernon, 1963; Grabert *et al.*, 1988; Hu *et al.*, 1992, 1993)) are generally nonlocal and complex, which is what gives rise to the noise and dissipation in the subdynamics. Moreover, since the slaving process generally involves a choice of causal initial conditions, an arrow of time (irreversibility) appears in the cloak of dissipation in the subdynamics (Mori, 1965; Nakajima, 1958; Willis and Picard, 1974; Zwanzig, 1960).

3.2. Quantum-Classical Aspect: Decoherence of Correlation History

Decoherence is brought on by the effect of a coarse-grained environment (or "irrelevant" sector) on the system (or the "relevant" sector). In the open systems philosophy, this split is imposed by hand, as when some of the fields, or the field values within a certain region of space-time, are chosen as relevant. This is not appropriate for treating close or nearly close systems which is what we have at hand. Instead, we adopt the consistent histories view (Griffiths, 1984; Omnés, 1988, 1990, 1992) of the quantum to classical transition problem, but instead of introducing projections along histories as in the decoherent histories scheme (Brun, 1993; Dowker and Halliwell, 1992; Dowker and Kent, 1995, 1996; Gell-Mann and Hartle, 1990, 1993; Halliwell, 1993, 1995, 1998; Hu *et al.*, 1993; Isham, 1994a; Isham *et al.*, 1998; Isham and Linden, 1994a; Kent, 1996, 1997, 1998; Paz and Sinha, 1991; Paz and Zurek, 1993; Twamley, 1993)[9] we let the correlation ordering in the full hierarchy act as its own projection. For any given degree of accuracy of observation or measurement which can be carried out on the close system there is a corresponding correlation order which is commensurate with it. What results

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[9] A closely related ongoing program is deducing hydrodynamic variables and dynamics (Anastopoulos, 1998; Brun and Halliwell, 1996; Brun and Hartle, 1999; Halliwell, 1998, 1999; Hartle *et al.*, 1995). It would be interesting to ask what the kinetic regime would be like in the decoherent histories scheme.

is an effectively open system described by an effective theory characteristic of the physics of the energy or length scale defined by the highest correlation order effective in the system. There is no need to select a priori a relevant sector within the theory. This is the main philosophy behind the decoherence of correlation histories (DCH) (Calzetta and Hu, 1993) approach. In this framework, decoherence occurs as a consequence of the fluctuations in the higher order correlations and results in a classical dissipative dynamics of the lower order correlations.

Applying the DCH scheme to interacting quantum fields we consider the full evolution of the field described by the Dyson-Schwinger hierarchy as a fine-grained history while histories where only a finite number of Wightman functions are freely specified (with all others slaved to them) are coarse-grained. We have shown that the finite effective actions obtained for the subsystems of lower order correlations are related to the decoherence functional between two such histories of correlations (Calzetta and Hu, 1993), its acquiring an imaginary part signifies the existence of noise which facilitates decoherence. Thus decoherence of correlation histories is a necessary condition for the relevance of the classical theory as a description of observable phenomena. It can be seen that if the classical theory which emerges from the quantum subdynamics is dissipative, then it must also be stochastic.[10] From our correlation history viewpoint, the stochasticity is in fact not confined to the field distributions—the correlation functions would become stochastic as well (Kac and Logan, 1976; Calzetta and Hu 2000).

Taking the truncated hierarchy as an effectively open system we can relate the imaginary part of the finite effective actions describing the truncated correlations to the autocorrelation of the stochastic sources, i.e., correlation noises. From the properties of the complete (unitary) field theory which constitutes the closed (untruncated) system, one can show that the imaginary part of the effective action is related to the nonlocal part of the real part of the effective action which depicts dissipation. This is the origin of the fluctuation–dissipation relations for nonequilibrium systems (Hu and Sinha, 1995). With this additional stochastic source which drives the classical fields and their correlation functions, we obtain a set of Boltzmann–Langevin equations.

We thus see the interplay of two major paradigms in nonequilibrium statistical mechanics: the Boltzmann–BBGKY and the Langevin–Fokker–Planck descriptions and the intimate connection between dissipation, fluctuations/noise, and decoherence (Gell-Mann and Hartle, 1993; Hu, 1991), now manifesting in the hierarchy of correlations which defines the graded subsystems.

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[10] Because the fundamental variables are quantum in nature, and therefore subject to fluctuations, a classical, dissipative dynamics would demand the accompaniment of stochastic sources in agreement with the "fluctuation–dissipation theorems," for, otherwise, the theory would permit unphysical phenomena as the damping away of zero-point fluctuations.

4. CORRELATIONS IN A QUANTUM FIELD AND BLACK HOLE INFORMATION PARADOX

We now give an example of how the idea of correlation hierarchy can be applied to understand certain unsettling phenomena in semiclassical gravity, such as the black hole information loss paradox. The use of kinetic field theory ideas to approach the black hole information loss paradox we need to understand three conceptual points:[11] was first proposed in (Hu, 1996b). Similar viewpoint can be found in (Anglin *et al.*, 1995; Holzhey and Wilczek, 1992; Wilczek, 1993).

We assume that the black hole and the quantum field with Hawking radiation together constitute a closed system. Even though the quantum field might be assumed to be free in the beginning, interaction still exists in its coupling with the black hole, especially when strong backreaction is included. We can model this complete system by an interacting quantum field. A particle-field system is a particular case of it. Of course a black hole is different from a particle. In this modeling, we will first explain how information is registered or "lost" in an interacting quantum field, then the distinct features of a black hole and finally, the information loss paradox of black hole systems.

(1) How does one characterize the information content of a quantum field (interacting, as a model for the black hole - quantum field closed system, with backreaction)?

(2) How does the information flow from one part of this closed system (hole) to another (field) and vice versa in the lifetime of the black hole? Does information really get "lost"? If yes, where has it gone? Can it be retrieved? If no, where does it reside?

(3) What is special about black hole radiation system as distinct from ordinary particle/field system?

4.1. Correlation Functions as Registrar and Correlation Dynamics as Flowmeter of Information in Quantum Fields

The set of correlation functions provides us with the means to register the information content of a quantum field. As mentioned earlier, the complete set of

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[11] The idea introduced in (Hu, 1996b) which is reproduced here was based on several components developed in varying depths since 1986: The development of the correlation dynamics in quantum fields formalism was done with Esteban Calzetta (Calzetta and Hu, 1995, 2000), that of viewing black hole radiance and inflation as exponential scaling (Hu, 1988, 1996c) was explored with Yuhong Zhang (Calzetta *et al.*, 2001; Hu, 1991a; Hu, 1993; Hu and Zhang, 1990) partly on the basis of work on critical phenomena done earlier with Denjoe O'Connor (Hu and O'Connor, 1987). For general background on this issue, see the review of Page (1993), which contains a comprehensive list of references till 1993, and Bekenstein (1994) more recently.

$n = \infty$ correlation (Wightman) functions carries the complete information about the quantum state of the field. A subset of it which defines the subsystem, such as the mean field and the two-point function, as is used in the ordinary description of (effective) field theory, carries only partial information. The missing information resides in the correlation noise, and manifests as dissipation in the subsystem dynamics. In this framework, the entropy of an incompletely determined quantum system is simply given by $S = -Tr\rho_{\text{red}} \ln \rho_{\text{red}}$, where the reduced density matrix of the subsystem (say, consisting of the lower correlation orders) is formed by integrating out the environmental variables (the higher correlation orders) after the hierarchy is truncated with causal factorization conditions.

While the set of correlation functions acts as a registrar of information of the quantum system, keeping track of how much information resides in what order, the dynamics of correlations as depicted by the hierarchy of equations of motion derived from the master effective action depicts the flow of information from one order to another, up or down or crisscrossing the hierarchy. Correlation dynamics has been proposed for the description of many body systems before (Balescu, 1975), and applied to molecular and plasma kinetics. In the light of the above theoretical description we see this scheme as a potentially powerful way to systemize quantum information, i.e., keeping track of the content and flow of information in a coherent or partially coherent quantum system.

4.2. Information Appears Lost to Subsystems of Lower Order Correlations— "missing" Information Stored in Higher Order Correlations

Most measurements of a quantum field system are of a local or quasilocal nature. If one counts the information content of a system on the basis of the mean field and the lowest order correlation functions, as in the conventional way (of defining quantum field theory in terms of, e.g., 2PI effective action), one would miss out a good portion of the information in the complete system, as much of that now resides in the higher order correlation functions in the hierarchy. These invoke nonlocal properties of the field, which are not easily accessible in the ordinary range of accuracy in measurements. Such an observer would then report a loss of information in his way of accounting (which is taken to be in agreement with other observers with the same level of accuracy of measurement). Only observers who have access to all orders (the "master" in the master effective action) would be able to see the complete development of the system and tell when the nth-order observer begins to lose track of the information count and report an information loss. This is more easily seen in molecular dynamics: For observers confined to measuring one particle distribution functions (truncation of the BBGKY hierarchy) and with the molecular chaos assumptions implicitly invoked (causal factorization condition, or "slaving"), he would report on information loss (not if it is a simple truncation with no slaving, as the subsystem will then be closed). This is how Boltzmann reasons

out the appearance of dissipation in ordinary macroscopic physical phenomena. The same can be said about measurement of quantum systems.

4.3. Exponential Scaling in Hawking Effect Facilitates Information Transfer to the Higher Correlations. Black Hole with Its Radiation Contains Full Information, but Retrieval Requires Probing the Higher Order Nonlocal Properties of the Field

How is this scheme useful in addressing the black hole information problem? How is the black hole/quantum field system different from the ordinary cases? The above scheme can explain the apparent loss of information in a quantum system, but there is an aspect distinct to black holes or systems emitting thermal radiance. It was observed that all mechanisms of emission of (coherent) thermal radiance such as the Hawking effect in black holes, or the Unruh effect in accelerated detectors, involve an exponential redshifting process in the system (Hu, 1988, 1996c). This can be compared to the scaling transformation in treating critical phenomena (Calzetta *et al.*, 2001; Hu, 1991a; Hu, 1993; Hu and Zhang, 1990). After sufficient exponential redshifting (at late times of collapse) and the black hole is emitting thermal radiance, the system has reached a state equivalent to the approach to a critical point in phase transition. There, the physical properties of the system are dominated by the infrared behavior, and as such, the lowest order correlation functions are no longer sufficient to characterize the critical phenomena. The contribution of higher order correlation functions would become important. Note that in ordinary situations, only the mean field and the two- or three-point correlation functions are needed to give an adequate description of the dynamics of the system. But for black holes or similar systems where exponential redshifting is at work, higher order correlations are readily activated. The information content profile for a quantum field in the presence of a black hole would be very different from ordinary systems, in that it is more heavily populated in the higher order correlations. If one carries out measurements which are only sensitive to the lower correlations, one would erroneously conclude that there is information loss.

So, following the correlation dynamics of the black hole/field system, while the state of the combined system remains the same as it had begun, there is a continuous shifting of information content from the black hole to the higher correlations in the field as it evolves. Correlation dynamics of fields can be used to keep track of this information flow. We speculate that the information content of the field will be seen to shift very rapidly from low correlation orders to the higher ones as Hawking radiation begins and continues. The end state of the system would have a black hole evaporated, and its information content transferred to the quantum field, with a significant portion of it residing in the higher order nonlocal correlations.

This is, however, not the end of the story for the correlation dynamics and information flow in the field. The information contained in the field will continue,

as it does in general situations, to shift across the hierarchy. As we know from the BBGKY description of molecular dynamics, after the higher correlation orders in the hierarchy have been populated—and for systems subjected to exponential redshifting this condition could be reached relatively quickly—the information will begin to trickle downwards in the hierarchy, though far slower than the other direction initially. The time it takes (with many crisscrossing) for the information to return to the original condition is the Poincare recurrence time. This time we suspect is the upper bound for the recoherence time, the time for a coherent quantum system interacting with some environment to regain its coherence (Anglin *et al.*, 1995). It would be interesting to work out the information flow using the correlation dynamics scheme for a few sample systems, both classical and quantum, so as to distinguish the competing effects of different characteristic processes in these systems, some quantum, some statistical (e.g., decoherence time, relaxation time, recoherence time, and recurrence time).[12]

5. FROM STOCHASTIC TO QUANTUM GRAVITY VIA METRIC CORRELATION HIERARCHY

After this expose of the notion of correlation dynamics (kinetic theory) representation of quantum field theory, and an example of its application to dealing with some conceptual issues, we now return to the main progression of ideas: What then? Where is quantum gravity?

5.1. Quantum Coherence From Correlations of Induced Metric Fluctuations

Let us take another look at the equations in the section where we compare the relation of semiclassical and the stochastic regimes with the quantum regime. We see first that in the semiclassical case, the classical metric correlations are given by the product of the vacuum expectation value of the energy momentum tensor whereas in the quantum case the quantum average of the correlation of metric (operators) is given by the quantum average with respect to the fluctuations in both the matter and the gravitational fields. In the stochastic case the form is closer to the quantum case except that now the quantum average is replaced by the noise

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[12] Our depiction above uses the interacting field model. Simpler cases might show somewhat degenerate behavior. An example is the interesting result of recoherence reported by Anglin *et al.* (1995). We think their reported result of a recoherence time of the order of the relaxation time is special to the simple model of particle free-field interaction. As the field modes couple only through their interaction with the particle, and not amongst themselves, there is no structure or dynamics of the information content of the field itself, and the only time scale for it to return is via interaction with the particle, which is why the recoherence time is related to the relaxation time of the particle. We expect in more general and complex systems (thus excluding many spin systems) the recoherence time is much longer than the relaxation time, more in the order of the recurrence time.

average, and the average of the energy momentum tensor is taken with respect only to the matter field. The important improvement over the semiclassical case is that it now carries information on the correlation of the energy momentum tensor of the fields and its induced metric fluctuations. This is another way to see why the stochastic description is closer to the quantum truth. More intuitively, the difference between quantum and semiclassical is that the latter loses all the coherence in the quantum gravity sector. Stochastic improves on the semiclassical situation in that partial information related to the coherence in the gravity sector is preserved as is reflected in the backreaction from the quantum fields and manifests as induced metric fluctuations. That is why we need to treat the noise terms with maximal respect. It contains quantum information absent in the classical. The coherence in the geometry is related to the coherence in the matter field, as the complete quantum description should be given by a coherent wave function of the combined matter and gravity sectors. Since the degree of coherence can be measured in terms of correlations, our strategy is to examine the higher correlations of the matter field, starting with the variance of the energy momentum tensor in order to probe into or retrieve whatever partial coherence remains in the quantum gravity sector. The noise we worked out in the ELE above contains the fourth-order correlation of the quantum field (or gravitons when considered as matter source) and manifests as induced metric fluctuations.

If we view classical gravity as an effective theory, i.e., the metric or connection functions as collective variables of the fundamental constituents which make up space-time in the large and general relativity as the hydrodynamic limit, we can also ask if there is a midway weighing station like kinetic theory of molecular dynamics, from quantum microdynamics to classical hydrodynamics. This transition involves both the micro-to-macro transition and the quantum-to-classical transition, which is what constitutes the mesoscopic regime for us.

5.2. Stochastic Boltzmann–Einstein Equations for Space-Time Correlations and Fluctuations

In the Introduction we have defined the master effective action and showed its relation to the Schwinger-Dyson hierarchy. From this one can establish a kinetic theory of nonlinear quantum fields (Calzetta and Hu, 1988). Here a brief description of the difference between an open system and an effectively open system is perhaps useful. While the Langevin equation describes an open system, its noise source arising from coarse-graining the environment, the full BBGKY hierarchy describes a closed system, coarse-graining (truncation and slaving) the hierarchy yields an effectively open system, for which Boltzmann's theory is the lowest order example. The Boltzmann equation describes the evolution of one particle distribution function driven by a two-particle collision integral with causal factorization condition (an example of slaving). Note that truncation without slaving will not

produce an open system, the nth-order correlation functions constitute a smaller closed system. They obey an equation of the Vlasov type which is unitary. Inclusion of a noise source representing the slaved contributions of the higher correlation functions gives the stochastic Boltzmann equation. The stochastic Boltzmann equation (Calzetta and Hu, 2000) contains features which would enable us to make connection with the stochastic equation in semiclassical gravity.

The hierarchical structure illustrated here for interacting quantum fields can be extended to a description of the microstructure of space-time—assuming that it can be represented by some interacting quantum field, or its extended version, string field theory. Starting with stochastic gravity we can get a handle on the correlations of the underlying field of space-time by examining (observationally if possible, e.g., effects of induced space-time fluctuations) the hierarchy of equations, of which the ELE is the lowest order, i.e., the relation of the mean field to the two-point function, and the two-point function to the four (variance in the energy momentum tensor), and so on. One can in principle move higher in this hierarchy to probe the dynamics of the higher correlations of space-time substructure. This addresses the correlation aspect; the quantum to classical aspect can be treated by the decoherence of correlation histories (Calzetta and Hu, 1993) scheme discussed in an earlier section.[13]

5.3. Strongly Correlated Systems: Space-Time Conductance Fluctuations

At this point it is perhaps useful to bring back another theme we explored in an earlier exposition of this subject matter, i.e., semiclassical gravity as mesoscopic physics.[14]

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[13] In (Hu, 1999) I also brought up the relevance of the large N expansion in gravity for comparison. There exists a relation between correlation order and the loop order (Calzetta and Hu, 1995). One can also relate it to the order in large N expansion (see, e.g., Aarts *et al.*, 2002; Asrts and Berges, 2002). It has been shown that the leading order $1/N$ expansion for an N-component quantum field yields the equivalent of semiclassical gravity (Hartle and Horowitz, 1981). The leading order $1/N$ approximation yields mean field dynamics of the Vlasov type which shows Landau damping which is intrisically different from the Boltzmann dissipation. In contrast the equation obtained from the nPI (with slaving) contains dissipation and fluctuations manifestly. It would be of interest to think about the relation between semiclassical and quantum in the light of the higher $1/N$ expansions (Tomboulis, 1977), which is quite different from the scenario associated with the correlation hierarchy.

[14] To practitioners in condensed matter and atomic/optical physics, mesoscopia refers to rather specific problems where, for example, the sample size is comparable to the probing scale (nanometers), or the interaction time is comparable to the time of measurement (femtosecond), or that the electron wave function correlated over the sample alters its transport properties, or that the fluctuation pattern is reproducible and sample specific. Take quantum transport. Traditional transport theory applied to macroscopic structures are based on kinetic theory while that for mesoscopic structures is usually based on near-equilibrium or linear response approximations (e.g., Landauer–Bütiker formula). New nanodevice operations involve nonlinear, fast-response, and far-from-equilibrium processes which are sensitive to the phases of the electronic wave function over the sample size. These necessitate a

Viewing the issues of correlations and quantum coherence in the light of mesoscopic physics we see that what appears on the right hand side of the ELE, the stress–energy two-point function, is analogous to conductance of electron transport which is given by the current–current two-point function. What this means is that we are really calculating the transport functions of the matter particles as depicted here by the quantum fields. Following Einstein's observation that space-time dynamics is determined by (while also dictates) the matter (energy density), we expect that the transport function represented by the current correlation in the fluctuations of the matter energy density would also have a geometric counterpart and equal significance at a higher energy than the semiclassical gravity scale. This is consistent with general relativity as hydrodynanics: conductivity, viscosity, and other transport functions are hydrodynamic quantities. Here we are after the transport functions associated with the dynamics of space-time structures. The Einstein tensor correlation function illustrated in an earlier section (Martin and Verdaguer, 1999a, 2000) is one such example. For many practical purposes we do not need to know the details of the fundamental constituents or their interactions to establish an adequate depiction of the low or medium energy physics, but can model them with semiphenomenological concepts like mean free path and collisional cross sections applicable at the lowest level (where correlation and coherence are ignored). In the mesoscopic domain the simplest kinetic model of transport using these concepts are no longer accurate enough. One needs to work with system–environment models and keep the phase information of the collective electron wave functions. When the interaction among the constituents gets stronger, or the probing scale gets shorter, effects associated with the higher correlation functions of the system begin to show up. Studies in strongly correlated systems are revealing in these regards. For example, conductance fluctuations obtained from the four-point function of the current carry important information such as the sample specific signature and universality. Although we are not quite in a position, technically speaking, to calculate the energy momentum four-point function (see, however, Shiokawa (2000)), thinking about the problem in this way may open up many interesting conceptual possibilities, e.g., what do universal conductance fluctuations mean for space-time and its underlying constituents? (In the same vein, I think studies of non-perturbative solutions of gravitational wave scattering in M(atrix) theory (Becker *et al.*, 1997) or exact solutions of colliding gravitational waves (see, e.g. Belinski and Verdaguer, 2001; Dorca and Verdaguer, 1993, 1994, 1997), the Khan and Penrose, 1971 or Nutku and Halil, 1977 solutions, and Misner's harmonic mapping theory (Misner, 1978) will also reveal interesting information about the underlying

new microscopic theory of quantum transport. One serious approach is using the Keldysh method in conjunction with Wigner functions. It is closely related to the CTP formalism we developed for nonequilibrium quantum fields aimed for similar problems in the early universe and black holes (Calzetta *et al.*, 2000; Calzetta and Hu, 1988).

structure of space-time beyond the hydrodynamic realm.) Thus, viewed in the light of mesoscopic physics, with stochastic gravity as a stepping stone, we can begin to probe into the higher correlations of quantum matter and with them the associated excitations of the collective modes in geometro-hydrodynamics, the kinetic theory of space-time mesodynamics and eventually quantum gravity—the theory of space-time microdynamics.

5.4. Task Summary

To summarize, our pathway from stochastic to quantum gravity is via the correlation hierarchy of induced metric fluctuations. This involves three essential tasks: (1) deduce the correlations of metric fluctuations from correlation noise in the matter field, (2) reconstitution of quantum coherence—this is the reverse of decoherence—from these correlation functions, and (3) use the BL equations to identify distinct collective variables depicting recognizable metastable structures in the kinetic and hydrodynamic regimes of quantum matter fields and how they demand of the corresponding space-time counterparts. This will give us a hierarchy of generalized stochastic equations—call them the Boltzmann–Einstein hierarchy for quantum gravity—for each level of space-time structure, from the macroscopic (general relativity) through the mesoscopic (stochastic gravity) to the microscopic (quantum gravity). These equations in the Boltzmann–Einstein hierarchy should be derivable from a master effective action of quantum gravity, a good challenge for the future.[15]

Before we close, let me add a remark that another route in connecting general relativity (read: hydrodynamics) to kinetic theory is via the covariant Wigner function in curved space-times. This was initiated by Calzetta *et al.* (1988; Calzetta and Hu, 1989; Kandrup, 1988) and pursued further by Antonsen (1997) and Fonarev (1994). It has the advantage that much work has been done in relating classical to quantum and relating hydrodynamics to kinetic theory via the Wigner function. The Wigner function obeys a Fokker–Planck equation; indeed a Wheeler–DeWitt–Vlasov equation for mini-superspace quantum cosmology has been obtained before (Calzetta and Hu, 1989). The kinetic theory formulation of quantum gravity via the Wigner function can be carried out in parallel to the Einstein–Langevin formulation presented here (if we talk about pure space-time structure without quantum fields, we can assume that the quantum field source is provided by gravitons), as the relation between a Langevin equation and a Fokker–Planck equation is well

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[15] This is not the Einstein–Boltzmann equation in classical general relativity and kinetic theory which frames the classical matter in the Boltzmann style as source of the Einstein equation. Our quantum or stochastic Boltzmann–Einstein equation refers to gravity and space-time alone. "Boltzmann" is to show the kinetic theory nature, and "Einstein" to show its space-time structure, albeit these two giants provide their respective theories only for the lowest correlation order and its dynamics: in distribution function of space-time and in geometro-hydrodynamics, respectively.

known. There is a hierarchy of Wigner functions corresponding to that of the correlation hierarchy discussed here. What needs to be done in the latter approach to bring it to a par with the former is to extract the noise term (see, e.g., Calzetta and Hu, 2000; Hu, unpublished) at the level of stochastic Boltzmann equation if we work with an interacting quantum field, or with the Boltzmann–Einstein hierarchy of quantum gravity if we refer to the hierarchy of the microstructure of space-time. The nature of our quest and the task to be performed are similar.

Now to the tasks: Indications of task (1) are exemplified by the recent work of Martin and Verdaguer (2000) where they derived for perturbations off Minkowski space the Einstein tensor correlation function. An example of task (3) is the metric conductance fluctuations explored in the work of Shiokawa (2000). Task (2), the reconstitution of quantum phase information from correlation functions in the field, is illustrated here with a suggestion based on the correlation hierarchy applied to the black hole information "loss" puzzle (Hu, 1996). We intend to return to this issue with the model of a relativistic particle moving in a quantum field (Hu, in preparation) based on the theses of Johnson (1999) and Raval (1996). Further discussions of hydrodynamic excitations of space-times can be found in (Hu, 2003). Last but not least, even though we focus here on the kinetic/hydrodynamic theory and noise/fluctuations aspects in seeking a clue to the microtheory of space-time from macroscopic constructs, another very important factor is topology. Topological features can have a better chance to survive the coarse-graining or effective/emergent processes to the macro world and can be a powerful key to unravel the microscopic mysteries.

To conclude, a simple message of this paper is that at our current level of understanding perhaps it is more fruitful or even correct to probe into the macro-to-micro aspect than simply quantizing the metric or connection form of general relativity.

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REFERENCES

Aarts, G., Ahrensmeier, D., Baier, R., Berges, J., Serreau, J. (2002). Far-from-equilibrium dynamics with broken symmetries from the 2PI-1/N expansion *Preprint* hep-ph/0201308.

Aarts, G. and Berges (2002). *Physical Review Letters* **88**, 041603.

Abe, Y., Ayik, A., Reinhard, P.-G., and Suraud, E. (1996). *Physics Reports* **275**, 49.

Adler, S. (1982). *Review of Modern Physics* **54**, 729.

Akhiezer, A. I. and Peletminsky, S. V. (1981). *Methods of Statistical Physics*, Pergamon, London.

Anastopoulos, C. (1998). *Classicality via Hydrodynamics in Quantum Field Theory*. *Preprint* gr-qc/9805074.

Anastopoulos, C. (2001). *Physical Review D: Particles and Fields* **63**, 125024. *Preprint* gr-qc/0011111.

Anglin, J. R., Laflamme, R., Zurek, W. H., and Paz, J. P. (1995). *Physical Review D* **52**, 2221.

Antonsen, F. (1997). *Physical Review D: Particles and Fields* **56**, 920.

Bakshi, P. M. and Mahanthappa, K. T. (1963a). *Journal of Mathematical Physics* **4**, 1.

Bakshi, P. M. and Mahanthappa, K. T. (1963b). *Journal of Mathematical Physics* **4**, 12.

Balescu, R. (1975). *Equilibrium and Nonequilibrium Statistical Mechanics*, Wiley, New York.

Barrabès, C., Frolov, V., and Parentani, R. (2000). *Physical Review D: Particles and Fields* **62**, 044020.

Becker, K., Becker, M., Polchinski, J., and Tseytlin, A. (1997). Higher order graviton scattering in m(atrix) theory. *Physical Review D: Particles and Fields* **56**, 3174–3178 *Preprint* hep-th/9706072.

Bekenstein, J. D. (1995). "Do we understand black hole entropy?" In *Proceedings of the 7th Marcel Grossmann Meeting on Recent Developments of General Relativity*, R. Ruffini, ed., Stanford University, July 1994. (World Scientific, Singapore) *Preprint* gr-qc/9409015.

Belinski, V., and Verdaguer, E. (2001). *Gravitational Solitons*, Cambridge University Press, Cambridge.

Bezzerides, B. and Du Bois, D. F. (1972). *Annals of Physics* (NY) **70**, 10.

Birrell, N. D. and Davies, P. C. W. (1982). *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England.

Blaizot, J.-P. and Iancu, E. (2001). The quark-gluon plasma: collective dynamics and hard thermal loops. Physics Reports. *Preprint* hep-ph/0101103.

Brun, T. (1993). *Physical Review D: Particles and Fields* **47**, 3383.

Brun, T. A. and Halliwell, J. J. (1996). *Physical Review D: Particles and Fields* **54**, 2899–2912.

Brun, T. A. and Hartle, J. B. (1999). *Physical Review D: Particles and Fields* **60**, 123503.

Boyanovsky, D., Lawrie, I., and Lee, D.-S. (1996). *Physical Review D: Particles and Fields* **54**, 4013.

Caldeira, A. O. and Leggett, A. J. (1983a). *Physica* **121A**, 587.

Caldeira, A. O. and Leggett, A. J. (1983b). *Annals of Physics* (NY) **149**, 374.

Calzetta, E. and Gonorazky, S. (1997). *Physical Review D: Particles and Fields* **55**, 1812.

Calzetta, E. and Hu, B. L. (1987). *Physical Review D: Particles and Fields* **35**, 495.

Calzetta, E., Habib, S. and Hu, B. L. (1988). *Physical Review D: Particles and Fields* **37**, 2901.

Calzetta, E., and Hu, B. L. (1989a). *Physical Review D: Particles and Fields* **40**, 380.

Calzetta, E., and Hu, B. L. (1989b). *Physical Review D: Particles and Fields* **40**, 656.

Calzetta, E., and Hu, B. L. (1993). Decoherence of correlation histories. In *Directions in General Relativity, Vol II: Brill Festschrift*, B. L., Hu, and T. A. Jacobson, eds. Cambridge University Press, Cambridge. *Preprint* gr-qc/9302013.

Calzetta, E. and Hu, B. L. (1994). *Physical Review D: Particles and Fields* **49**, 6636.

Calzetta, E. and Hu, B. L. (1995a). *Physical Review D: Particles and Fields* **52**, 6770.

Calzetta, E., and Hu, B. L. (1995b). Correlations, decoherence, disspation and noise in quantum field theory. In *Heat Kernel Techniques and Quantum Gravity*, Fulling, S. ed., Texas A&M Press, College Station. *Preprint* hep-th/9501040.

Calzetta, E. Hu, B. L. (1999). *Physical Review D: Particles and Fields* **59** *Preprint* 065018 quant-ph/9809084.

Calzetta, E. and Hu, B. L. (2000). *Physical Review D: Particles and Fields* **61**, 025012.

Calzetta, E., and Hu, B. L. (1988). *Physical Review D: Particles and Fields* **37**, 2878.

Calzetta, E., Hu, B.-L., and Mazzitelli, F. D. (2001a). *Physics Report* **352**, 459–520.

Calzetta, E., Hu, B.-L., and Mazzitelli, F. D. (2001b). *Coarse-Grained Effective Action and Renormalization Group Theory in Semiclassical Gravity and Cosmology Preprint* hep-th/0102199.

Calzetta, E., Hu, B. L., Ramsey, S. (2000). *Physical Review D: Particles and Fields* **61**, 125013.

Calzetta, E. Roura, A. and Verdaguer, E. (2001). *Physical Review D: Particles and Fields Preprint* quant-ph/0011097.

Campos, A. and Hu, B. L. (1998). *Physical Review D: Particles and Fields* **58**, 125021.

Campos, A. and Hu, B. L. (1999). *International Journal of Theoretical Physics* **38**, 1253.

Campos, A. and Verdaguer, E. (1994). *Physical Review D: Particles and Fields* **49**, 1861.

Campos, A. and Verdaguer, E. (1996). *Physical Review D: Particles and Fields* **53**, 1927.

Candelas, P. and Sciama, D. W. (1977). *Physics Review Letters* **38**, 1372.

Carlip, S. (1997). *Physics Review Letters* **79**, 4071.

Carlip, S. (1998). *Classical Quantum Gravity* **15**, 2629.

Danielewicz, P. (1984). *Annals of Physics* (NY) **152**, 239.

DeWitt, B. S. (1975). *Physics Reports* **19C**, 297.

DeWitt, B. S. (1986). In *Quantum Concepts in Space and Time*, R. Penrose and C. J. Isham eds., (Clarendon Press, Oxford.

Dorca, M. and Verdaguer, E. (1993). *Nuclear Physics B* **403**, 770.

Dorca, M. and Verdaguer, E. (1994). *Physical Review D: Particles and Fields* **50**, 2631.

Dorca, M. and Verdaguer, E. (1997). *Nuclear Physics B* **484**, 435.

Dowker, H. F. and Halliwell, J. J. (1992). *Physical Review D: Particles and Fields* **46**, 1580

Dowker, F. and Kent, A. (1995). *Physics Review Letters* **75**, 3038.

Dowker, F. and Kent, A. (1996). *Journal of Statistical Physics* **82**, 1575.

Du Bois, D. F. (1967). In *Lectures in Theoretical Physics Vol 9c*, W. Brittin, ed., Gordon and Breach, New York.

Feynman, R. and Vernon, F. (1963). *Annals of Physics* (NY) **24**, 118.

Feynman, R. and Hibbs, A. (1965). *Quantum Mechanics and Path Integrals*, McGraw–Hill, New York.

Fonarev, O. (1994). *Journal of Mathematical Physics* **35**, 2105.

Ford, L. H. and Svaiter, N. F. (1997). *Physical Review D: Particles and Fields.* **56**, 2226.

Ford, L. H. (1982). *Annal of Physics* **144**, 238.

Fulling, S. A. (1989). *Aspects of Quantum Field Theory in Curved Space-Time*, Cambridge University Press, Cambridge, England.

Garay, L. J. (1998a). *Physics Review Letters* **80**, 2508.

Garay, L. J. (1998b). *Physical Review D: Particles and Fields* **58**, 124015.

Garay, L. J. (1999). *International Journal of Modern Physics A* **14**, 4079.

Gell-Mann, M. and Hartle, J. B. (1990). In *Complexity, Entropy and the Physics of Information*, W. H. Zurek, ed., Addison-Wesley, Reading.

Gell-Mann, M. and Hartle, J. B. (1993). *Physical Review D: Particles and Fields* **47**, 3345.

Grabert, H. (1982). *Projection Operator Techniques in Non Equilibrium Statistical Mechanics*, Springer-Verlag, Berlin.

Grabert, H., Schramm, P., and Ingold, G. L. (1988). *Physical Report* **168**, 115.

Grib, A. A., Mamayev, S. G., and Mostepanenko, V. M. (1994). *Vacuum Quantum Effects in Strong Fields*, Friedmann Laboratory, St. Petersburg, FL.

Griffiths, R. B. (1984). *Journal of Statistical Physics* **36**, 219.

Haag, R. (1992). *Local Quantum Physics*, Springer, Berlin.

Halliwell, J. J. (1993). *Physical Review D: Particles and Fields* **48**, 4785.

Halliwell, J. J. (1995). *Annals of New York Academic Sciences* **755**, 726 [review].

Halliwell, J. J. (1998b). *Physical Review D: Particles and Fields* **57**, 2337–2348.

Halliwell, J. J. (1998c). *Physical Review D: Particles and Fields* **58**, 105015.

Halliwell, J. J. (1999). *Physics Review Letters* **83**, 2481.

Hartle, J. B. (1993). Quantum mechanics of closed systems. In *Directions in General Relativity*, I, B. L. Hu, M. P. Ryan, and C. V. Vishveswara. eds., (Cambridge University, Cambridge.

Hartle, J. B. and Gell- Mann, M. (● ● ●). *Physical Review D: Particles and Fields* **47**, 3345.

Hartle, J. B. and Horowitz, G. T. (1981). *Physical Review D: Particles and Fields* **24**, 257.

Henning, P. (1995a). *Physics Reports* **253**, 235.

Henning, P. (1995b). *Nuclear Physics A* **582**, 633.

Hartle, J. B., Laflamme, R., Marolf, D. (1995c). *Physical Review D: Particles and Fields* **51**, 7007.

Holzhey, C. and Wilczek, F. (1992). *Nuclear Physics B* **380**, 447.

Hu, B. L. (1988). In *Proceedings of the CAP-NSERC Summer Institute in Theoretical Physics, Vol 2*, K. Khanna, G. Kunstatter, and H. Umezawa, eds., Edmonton, Canada, July 1987. (World Scientific, Singapore, 1988)

Hu, B. L. (1989). *Physica A* **158**, 399.

Hu, B. L. (1991a). Coarse-graining and backreaction in inflationary and minisuperspace cosmology. In *Relativity and Gravitation: Classical and Quantum*, J. C. D'Olivo *et al.* eds., Proceedings SILARG VII, Cocoyoc, Mexico. (World Scientific, Singapore 1991)

Hu, B. L. (1991b). Statistical mechanics and quantum cosmology. In *Proceedings of Second International Workshop on Thermal Fields and Their Applications*, H. Ezawa *et al.*, eds., North-Holland, Amsterdam.

Hu, B. L. (1993). *Classical Quantum Gravity* **10**, S93.

Hu, B. L. (1994). Quantum statistical fields in gravitation and cosmology. In *Proceedings Third International Workshop on Thermal Field Theory and Applications*, R. Kobes and G. Kunstatter, eds., World Scientific, Singapore. *Preprint* gr-qc/9403061.

Hu, B. L. (1996a). (unpublished) General Relativity as Geometro-hydrodynamics. Invited talk at the second International Sakharov Conference, Moscow, May, 1996. Preprint gr-qc/9607070.

Hu, B. L. (1996b). Correlation dynamics of quantum fields and black hole. In *Erice 1995, String Gravity and Physics at the Planck Energy Scale*, N. Sanchez and Zichichi, eds., Kluwer, Dordrecht, The Netherlands, pp 219–232. *Preprint* gr-qc/9511075.

Hu, B. L. (1996c). In *Proceedings of the Fourth International Workshop on Thermal Field Theory*, Y. X. Gui and K. Khanna, eds., Dalian, China, August 1995. (World Scientific, Singapore, 1996)

Hu, B. L. (1997). Semiclassical Gravity and Mesoscopic Physics. In *Quantum Classical Correspondence*, Feng, D. S., and Hu, B. L. eds., International Press, Boston. *Preprint* gr-qc/9511077.

Hu, B. L. (1999). "Stochastic Gravity" *International Journal of Theoretical Physics* **38**, 2987 *Preprint* gr-qc/9902064.

Hu, B. L. (2003). *What Can BEC Tell Us About Quantum Gravity?*, Invited talk at the 7th Peyresq Meeting, June, 2002. *International Journal of Theoretical Physics*.

Hu, B. L. (unpublished). *Noise From Quantum Field Theory* Talk given at the Workshop on Quantum Field theory of Nonequilibrium Processes at the Institute for Nuclear Theory , University of Washington, Seattle, Nov. 1999.

Hu, B. L., Johnson, P. and Roura, A. (in preparation).

Hu, B. L. and Matacz, A. (1995). *Physical Review D: Particles and Fields* **51**, 1577.

Hu, B. L. and O'Connor, D. J. (1987). *Physical Review D: Particles and Fields* **36**, 1701.

Hu, B. L., Paz, J. P., and Zhang, Y. (1992). *Physical Review D: Particles and Fields* **45**, 2843.

Hu, B. L., Paz, J. P., and Zhang, Y. (1993). *Physical Review D: Particles and Fields* **47**, 1576.

Hu, B. L. and Phillips, N. G. (2000). *International Journal of Theoretical Physics* **39**, 1817 *Preprint* gr-qc/0004006.

Hu, B. L. Raval, A., and Sinha, S. (1998). Notes on black hole fluctuations and backreaction. In *Black Holes, Gravitational Radiation and the Universe: Essays in Honor of C. V. Vishveshwara*, B. Iyer and B. Bhawal, eds., Kluwer Academic Publishers, Dordrecht, the Netherlands, *Preprint* gr-qc/9901010.

Hu, B. L. and Shiokawa, K. (1998a). *Physical Review D: Particles and Fields* **57**, 3474.

Hu, B. L. and Sinha, S. (1995). *Physical Review D: Particles and Fields* **51**, 1587.

Hu, B. L. and Zhang, Y. (1990). Coarse-graining, scaling, and inflation, University Maryland Preprint 90–186.

Hu, B. L. and Verdaguer, E. (2001). Recent advances in stochastic gravity: Theory and issues. In *Advances in the Interplay Between Quantum and Gravity Physics*, Bergmann, P., and De Sabbata, V., (Kluwer, Dordrecht, the Netherlands. 2002) *Preprint* gr-qc/0110.

Halliwell, J. J. (1998). *Physical Review D: Particles and Fields* **57**, 2337.

Isham, C. J. (1994a). *Journal of Mathematical Physics* **35**, 2157.

Isham, C. J. and Linden, N. (1994b). *Journal of Mathematical Physics* **35**, 5452.

Isham, C. J. and Linden, N. (1994c). *Journal of Mathematical Physics* **36**, 5392.

Isham, C. J., Linden, N., Savvidou, K., and Schreckenberg, S. (1998). *Journal of Mathematical Physics* **39**, 1818.

Jacobson, T. A. (1995). *Physics Review Letters* **75**, 1260–1263

Johnson, P. (1999). PhD Thesis, University of Maryland. Nonequilibrium Dynamics of Particle Field Interaction.

Johnson, P. R. and Hu, B. L. (2002). Stochastic theory of relativistic particles moving in a quantum field: Scalar Abraham–Lorentz–Dirac–Langevin equation, radiation reaction and vacuum fluctuations. *Physical Review D: Particles and Fields* **65** *Preprint* quant-ph/0101001.

Jordan, R. D. (1986). *Physical Review D: Particles and Fields* **33**, 44.

Kac, M. and Logan, J. (1976). *Physical Review A* **13**, 458.

Kac, M. and Logan, J. (1979). Fluctuations. In *Fluctuation Phenomena*, E. W. Montroll and J. L. Lebowitz, eds., Elsevier, New York, p. 1.

Kadanoff, L. and Baym, G. (1962). *Quantum Statistical Mechanics*, Benjamin, New York.

Kandrup, H. E., (1988). *Physical Review D: Particles and Fields* **37** 2165.

Keldysh, L. V. (1964). *Zhurnal Eksperimental' noi i Theoreticheskoi Fiziki* **47**, 1515 [English translation, (1965). *Soviet Physics JEPT* **20**, 1018.]

Kent, A. (1996). *Physical Review A* **54**, 4670.

Kent, A. (1997). *Physics Review Letters* **78**, 2874.

Kent, A. (1998). *Physics Review Letters* **81**, 1982.

Khan, K. (1971). *Nature* **222**, 185.

Kuo, C.-I. and Ford, L. H. (1993). *Physical Review D: Particles and Fields* **47**, 4510.

Li, S.-P. and McLerran, L. (1983). *Nuclear Physics B* **214**, 417.

Lombardo, F. C., and Mazzitelli, F. D. (1997). *Physical Review D: Particles and Fields* **55**, 3889.

Martin, R. and Verdaguer, E. (1999a). *Physical Review D: Particles and Fields* **60**, 084008.

Martin, R. and Verdaguer, E. (1999b). *Physics Letters B* **465**, 113.

Martin, R. and Verdaguer, E. (2000). *Physical Review D: Particles and Fields* **61**, 124024.

Massar, S. and Parentani, R. (2000). *Nuclear Physics B* **575**, 333.

Matacz, A. (1997). *Physical Review D: Particles and Fields* **55**, 1860.

Mirzabekian, A. G. and Vilkovisky, G. A. (1998). Particle creation in the effective action method. *Annals of Physics* 270 391–496. Preprint gr-qc/9803006.

Misner, C. W. (1978). *Physical Review D: Particles and Fields* **18**, 4510–4524.

Mori, H. (1965). *Progress of Theoretical Physics* **33**, 1338.

Mottola, E. (1986). *Physical Review D: Particles and Fields* **33**, 2136.

Mrowczynski, St. (1997). *Physical Review D: Particles and Fields* **56**, 2265.

Mrowczynski, St. and Danielewicz, P. (1990). *Nuclear Physics B* **342**, 345.

Mrowczynski, St. and Heinz, U. (1994). *Annals of Physics* **229**, 1.

Martin, R. and Verdaguer, E. (2000). *Physical Review D: Particles and Fields* **61**, 124024.

Nakajima, S. (1958). *Progress of Theoretical Physics* **20**, 948.

Niemeyer, J. C. and Parentani, R. (2001). *Preprint* astro-ph/0101451.

Nutku, Y. and Halil, M. (1977). *Physical Review Letters* **39**, 1379.

Omnés, R. (1988). *Journal of Statistical Physics* **53**, 893, 933, 957.

Omnés, R. (1990). *Annals of Physics* (N. Y.) **201**, 354.

Omnés, R. (1992). *Review of Modern Physics* **64**, 339.

Omnés, R. (1994). *The Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, NJ.

Page, D. N. (1994). "Black Hole Information" In *Proceedings of the 5th Canadian Conference on General Relativity and Relativistic Astrophysics*, R. B. Mann and R. G. McLenaghan, eds., University of Waterloo, May 1993. (World Scientific, Singapore) *Preprint* hep-th/9305040.

Paz, J. P. and Sinha, S. (1991). *Physical Review D: Particles and Fields* **44**, 1038.

Parentani, R. (2001). *Physical Review D: Particles and Fields* **63**, 041503.

Phillips, N. G. and Hu, B.-L. (1997). *Physical Review D: Particles and Fields* **55**, 6123.

Paz, J. P. and Zurek, W. H. (1993). *Physical Review D: Particles and Fields* **48**, 2728.

Phillips, N. G. and Hu, B. L. (2000). *Physical Review D: Particles and Fields* **62**, 084017.

Phillips, N. G. and Hu, B. L. (2001). *Physical Review D: Particles and Fields* **63**, 104001.

Raval, A. (1996). PhD. Thesis, University of Maryland. "Stochastic Properties of Particle Detector and Quantum Field Interaction"

Roura, A. and Verdaguer, E. (1999). *International Journal of Theoretical Physics* **38**, 3123.

Roura, A. and Verdaguer, E. (2000). *International Journal of Theoretical Physics* **39**, 1831.

Sakharov, A. D. (1968). *Soviet Physics Dokl.* 12 1040.

Schwinger, J. (1961). *Journal of Mathematical Physics* **2**, 407.

Shiokawa, K. (2000). *Physical Review D: Particles and Fields* **62**, 024002.

Sinha, S., Raval, A., and Hu, B. L. (2003). " Black hole fluctuations and backreaction in Stochastic Gravity" In *Thirty Years of Black Hole Physics* Foundation of Physics, Special Issue.

Sorkin, R. (1996). How wrinkled is the surface of a black hole? In *Proceedings of the First Australian Conference on General Relativity and Gravitation* David Wiltshire, ed., University of Adelaide, Adelaide, Australia, February 1999, pp. 163–174 *Preprint* gr-qc/9701056.

Sorkin, R. D. and Sudarsky, D. (1999). *Classical Quantum Gravity* **16**, 3835.

Spohn, H. (1991). *Large Scale Dynamics of Interacting Particles*, Springer-Verlag, Berlin.

Su, Z., Chen, L. Y., Yu, X., and Chou, K. (1988). *Physics Review B: Condensed Matter* **37**, 9810.

Schreckenberg, S. (1998). *Journal of Mathematical Physics* **39**, 1818.

Twamley, J. (1993). *Physical Review D: Particles and Fields* **48**, 5730.

Tomboulis, E. (1977). *Physical Letters B* **70**, 361.

van Leeuwen, W. A., van Weert, Ch. G., and de Groot, S. R. (1980). *Relativistic Kinetic Theory* North-Holland, Amsterdam.

Wald, R. M. (1994). *Quantum Field Theory in Curved Space Time and Black Hole Thermodynamics*, The University of Chicago Press, Chicago.

Wilczek, F. (1993). In *Black Holes, Membranes, Wormholes and Superstrings*, K. Kalara and D. V. Nanopoulos, eds., (World Scientific, Singapore)

Witte, F. M. C., and Klevansky, S. P. (1995). $\lambda\Phi^4$ nonequilibrium dynamics and kinetic field theory" *Preprint* hep-th/9501025.

Willis, C. R. and Picard, R. H. (1974). *Physical Review A* **9**, 1343.

Wheeler, J. A. (1962). *Geometrodynamics*, Academic Press, New York.

Wu, C.-H. and Ford, L. H. (1999). *Physical Review D: Particles and Fields* **60**, 104013.

Zhou, G., Su, Z., Hao, B., and Yu, L. (1985). *Physics Reports* **118**, 1.

Zhuang, P. and Heinz, U. (1998). *Physical Review D: Particles and Fields* **57**, 6525.

Zwanzig, R. (1960). *Journal of Chemical Physics* **33**, 1338.

Zwanzig, R. (1961). In *Lectures in Theoretical Physics* III, W. E. Britten, B. W. Downes, and J. Downs, eds., Interscience, New York, pp. 106–141.